Traffic Modeling and Optimization at Mountview Middle School

A Major Qualifying Project Report

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by

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Abstract

Traffic congestion is a problem of increasing concern and prevalence in today's society, and as a result, many people and organizations are making efforts to find ways to alleviate traffic congestion. Mathematical modeling is one method that can be employed to better understand traffic. Mountview Middle School (MTV), in Holden, Massachusetts, is an ideal subject for a study on modeling traffic flow using mathematical techniques. The school is one isolated example of a location that needs innovative solutions to lessen the excessive traffic congestion it faces on a daily basis, and the school's proximity to WPI makes it easy for us to collect data and communicate with policymakers of the school and of the town.

The focus of our study was the parent drop-off area at MTV. The short driveway on the school's campus, along with a drop-off area that is not optimally designed, lead to long lines of parents that often spill over onto the main road. We collected real-world data to model the situation using three different queuing models: a single-server model, a multi-server model, and a bulk service model. We used MATLAB to create a simulation that represented each of the queuing models. We also represented the drop-off situation as a Markov process by creating a transition matrix of five different states. Finally, we explored various solutions for redesigning the drop-off area by changing parameters of the system and seeking suggestions from traffic engineers in the area.

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Chapter 1

Introduction

Anyone who has used any form of transportation has had to deal with the nuisances of traffic congestion. Wasted time and late arrivals are the obvious negative impacts of traffic, but many more exist. According to the Texas Transportation Institute, America's 75 largest metropolitan areas experienced, "3.6 billion vehicle-hours of delay, resulting in 5.7 billion U.S. gallons in wasted fuel and $67.5 billion in lost productivity" in 2000. However, loss in money and productivity is not a problem isolated to large cities; while drivers in very large cities lose $1000 annually, it is estimated that drivers in rural areas lose $200 annually as a result of traffic congestion [4].

Traffic congestion is a problem of increasing concern and prevalence in today's society, and as a result, many people and organizations are making efforts to find ways to alleviate traffic congestion. Mathematical modeling is one method that can be employed to better understand traffic.

To study traffic from a mathematical perspective, it is easiest to start with a small-scale problem. Mountview Middle School (MTV), in Holden, Massachusetts, is an ideal subject for a study on modeling traffic flow using mathematical techniques. MTV is an isolated example of a location that needs innovative solutions to lessen the excessive traffic congestion it faces on a daily basis. The school's proximity to WPI makes it easy for us to collect data and also gives us the capability of communicating with policymakers of the school and of the town. This ideal combination of close proximity and access to policymakers gives us the unique advantage of being able to see the project through to completion: we can collect data, model the traffic flow, and make recommendations for new policies to help alleviate the congestion.

Mountview is situated on Shrewsbury Street in Holden, Massachusetts (see Figure 1.1 for a Google Maps image of MTV and the surrounding area).

Shrewsbury Street is used as a commuter route for out-of-towners and other

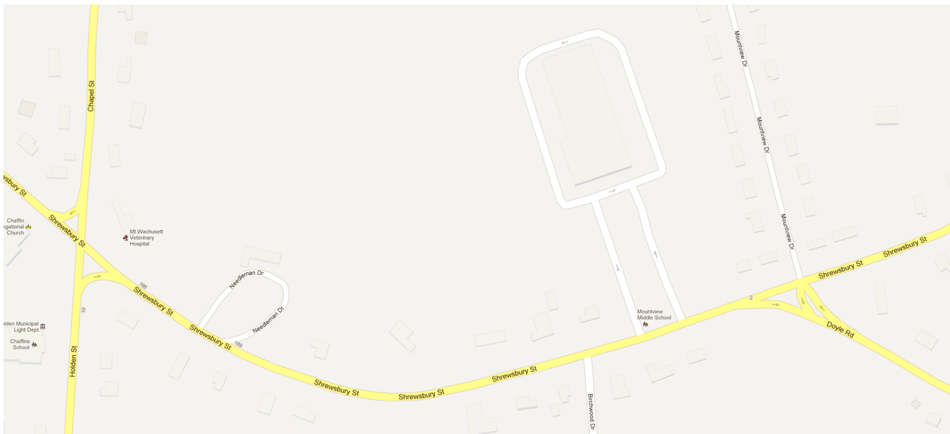

Figure 1.1: MTV and Surrounding Area

drivers trying to get to the nearby highway exit for Interstate I-190, located off of Doyle Road. Traffic signals are positioned at Doyle and at Holden/Chapel Street, flanking the school along Shrewsbury Street. In addition to commuter traffic, Mountview traffic includes parents who choose to drive their children to school every day. The short driveway on the school's campus, along with a parent drop-off area that is not optimally designed, lead to long lines of parents that often spill over onto Shrewsbury Street. This spillover prevents commuters from passing by the school, which in turn causes excessive queues on Shrewsbury Street. This compounding of queues makes driving near the school in the morning a headache for parents, employees, and commuters.

If we can understand the traffic dynamics at MTV, then we can begin to understand traffic problems of greater magnitude. The goal of our project is to model and improve the parent drop-off process at Mountview. In this paper, we will use queuing theory and markov chains to model the traffic patterns of parents at Mountview Middle School. We will also discuss the simulation we created to illustrate the queue of parents and the spillover onto Shrewsbury Street. Afterwards, we will discuss various solutions for redesigning the drop-off area at the school that could potentially alleviate some of the traffic congestion.

Chapter 2

A Close Look at MTV

In order to effectively model the traffic flow at Mountview, we need to study the school and be familiar with all components of the school and the drop-off process (see Figure 2).

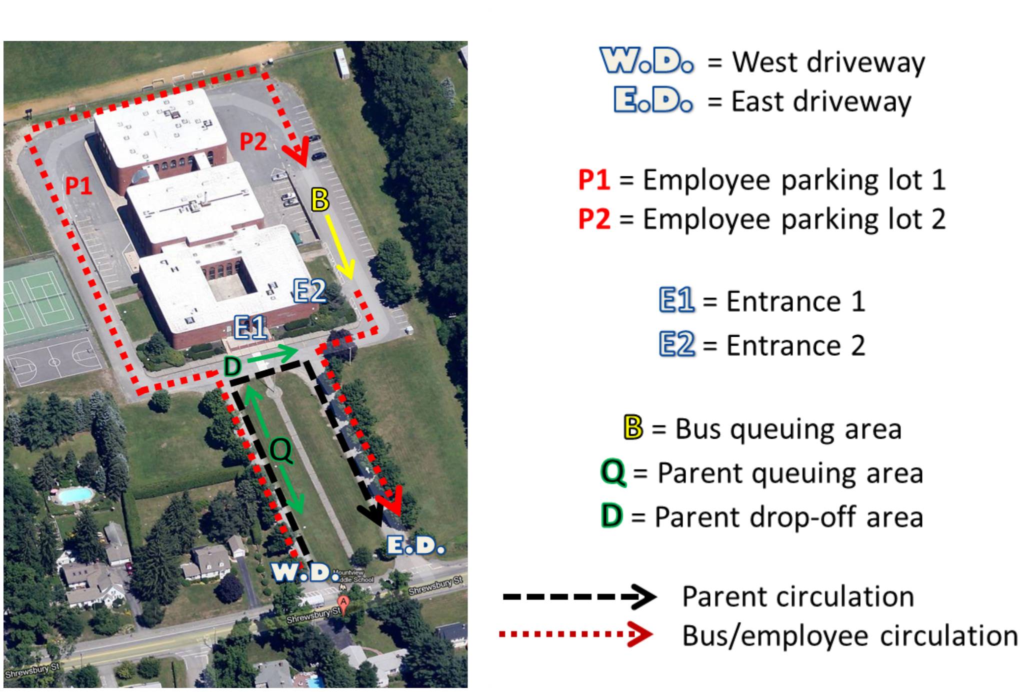

Figure 2.1: Traffic Layout at MTV

Mountview is situated on Shrewsbury Street in Holden, Massachusetts. Parents, employees, and buses enter the school's property via the school's West Driveway, $W.D.$, and exit via the school's East Driveway, $E.D.$ This flow of traffic from left to right is an exception to standard American driving conventions. Parents enter the school property and wait to drop off their children by forming a queue in the area denoted by the Q, which is located in the right lane of $W.D.$ They pull up in front of the school's main entrance $E1$ and drop off in the area denoted by D. Students who walk to school use the crosswalk located on Shrewsbury Street or one of the crosswalks on $W.D.$ or $E.D.$ to get to the pathway on the school's green (see Figure 2.2 for a picture of the pathway). After the students walk up the pathway and reach the end of the green, they use the crosswalk which cuts through D to enter the school through $E1$ (see Figure 2.3 for a picture of the crosswalk through the drop-off area).

Figure 2.2: Student Pathway on the School's Green

Buses and employees enter the school grounds and stay in the left lane of $W.D.$ in order to pass the parent drop-off area. The employees park in the parking lots $P1$ and $P2$, which surround the school, while the buses drive around the school to line up in the area denoted by B and drop off in front

Figure 2.3: Crosswalk through the Drop-Off Area

of the cafeteria entrance $E2$.

The crosswalk that cuts across the drop-off area often results in additional traffic congestion. If a child approaches the crosswalk when parents are in the midst of dropping off their children, parents will wait until the child uses the crosswalk before leaving the drop-off area. If a child starts to use the crosswalk when the drop-off area is empty, parents who approach the area willl stop at the edge of the crosswalk to unload. Since there is only enough space for one or two cars to drop off in the area before the crosswalk, a child in the crosswalk decreases the potential number of parents that could be dropping off at once.

The drop-off area for Mountview was originally designed to support a capacity of 20-40 parents dropping off. Currently, there are between 160 and 180 parents dropping off their kids on a daily basis. The original design of the school was not intended to support the high number of parents that currently it currently needs to support. The short driveway on the school's campus, along with a drop-off area that is not optimally designed, lead to long lines of parents that often spill over onto Shrewsbury Street. This spillover prevents commuters from passing by the school, which in turn causes excessive queues on Shrewsbury Street.

Chapter 3

Queuing Theory

Informally, queuing theory is the study of waiting lines, a situation where customers arrive at a service facility and must wait for service. The person or object performing the service is known as a server, and we will denote the number of servers in the system by s. We will denote the average arrival rate of customers in a unit time period by λ, and the mean number of customers served in a unit time period by μ. All of these parameters depend on the queueing discipline. Once the queuing discipline is known, we can determine all average-case operating characteristics of our system.

Different waiting lines may (and usually) have unique arrival rates, service rates, and number of servers [9].

3.1 Service System Characteristics

3.1.1 Arrival Characteristics

The three major arrival characteristics are size, behavior, and arrival distribution. The population size is either unlimited or limited. The population size is considered unlimited (or infinite) when the arrivals are only a fraction of the potential arrivals. The population size is considered limited (or finite) when the number of potential customers is countable.

The behavior of an arriving customer depends on the patience of that customer. When a customer arrives at a system and decides not to join the line because it is too long, the customer is *balking*. When a customer waits in line and decides to leave the line, the customer is *reneging*. When a customer is in a system with multiple queues and switches to a shorter queue, this customer is *jockeying*. To simplify the modeling process, most queuing models assume that customers are patient; once a customer arrives at a system, they stay in the system until they have been served and do not switch lines. For example, parents do not balk or renege at MTV; once parents arrive at the school, they only leave once they have dropped off their child. While there is only one queue, parents occasionally 'jockey' by trying to cut in front of other parents to drop off their kids (in other words, parents are jockeying to a new, shorter queue they have made by cutting the other parents).

The pattern of arrivals to the system can either be scheduled or random. An example of scheduled arrivals is a doctor's office, when each patient arrives at a specific time. Arrivals are random when they cannot be predicted exactly and when arrivals are independent of each other. The number of random arrivals is commonly estimated using a Poisson probability distribution. This distribution models the probability of a certain number of arrivals in a given unit time period by the following formula:

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

where

$x =$ number of arrivals per unit of time

$P(x) =$ probability of x arrivals

$\lambda =$ average arrival rate

3.1.2 Service Characteristics

The service characteristics include the design of the service system and the distribution of service times. A service system can be either a single-channel or a multiple-channel queuing system. In a single-channel system, there is one server, while in a multiple-channel system there are multiple servers. A service system can also be either single-phase or multi-phase. In a single-phase system, there is one stop for service. In a multi-phase system, there are multiple stops for service. Figure 3.1.2 shows the various combinations of channels and phases for a service system.

Service times can either be constant or random. If service times are constant, then every customer is served in the same amount of time. For example, customers at an automatic car wash will all be served in the same amount of time. If service times are random, then the distribution is usually described by the exponential probability distribution. For example,

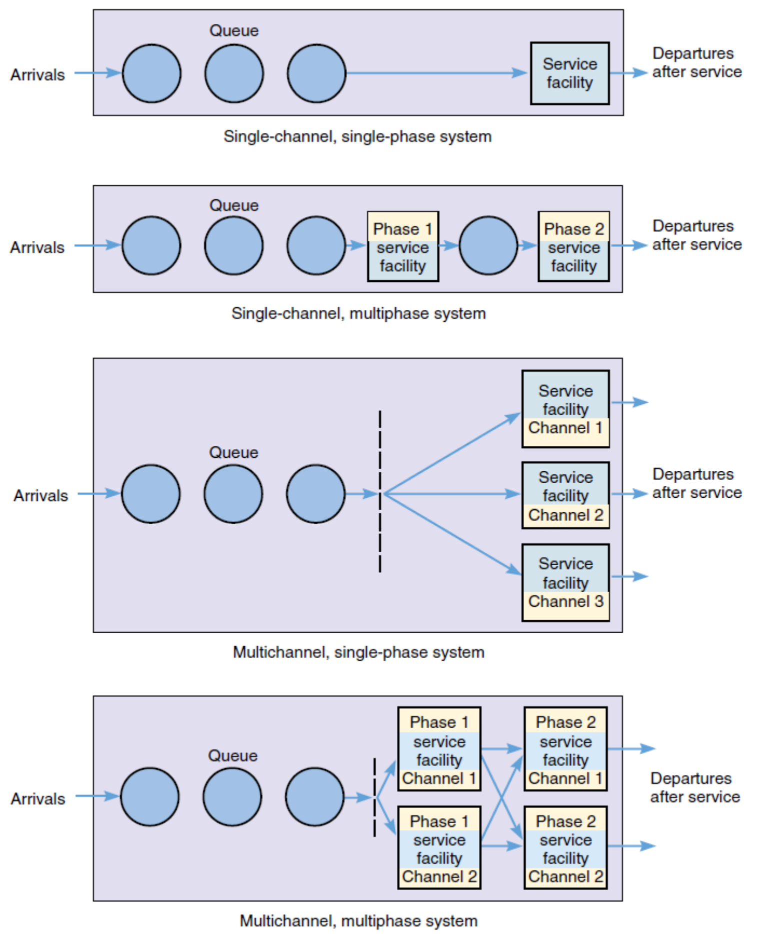

Figure 3.1: Queuing System Designs [9]

customers at a coffee shop wait different amounts of time to receive their orders.

Mountview can be viewed as a multi-phase system. Parents must wait on Shrewsbury Street for an opening in traffic in order to enter the West Driveway (phase 1). Once they have entered the driveway, they wait their turn to drop off their child (phase 2). After they drop off their children, parents wait at the end of the East Driveway for an opening in traffic that will allow them to reenter Shrewsbury Street. The service rate is random; parents enter WD, drop off their kids, and leave MTV via ED all at variable times.

3.1.3 Queue Characteristics

A queue varies by the maximum length of the line and its discipline. A queue length can either be limited or unlimited. The queue discipline describes the order of service for customers. Service can either be FIFO, LIFO, SIRO, or Priority. FIFO service stands for First-In, First-Out. In an FIFO system, customers are served in the order that they arrive. LIFO stands for Last-In, First-Out. In an LIFO system, a customer that has arrived most recently will be served first. SIRO stands for Service In Random Order. In a SIRO system, customers are served randomly. In a priority queue, customers are divided into priority classes and served based on their class [5].

3.2 Various Server Models

There are many different server models, but in this section we will discuss the single-server model, multi-server model, and the bulk service model. Recall the variables for a queuing model, which we will use to describe the single-server and multi-server models [2]:

μ = average number of customers who complete their service per unit time period
λ = average number of customers arriving per unit time period
s = number of servers in the system
L = average number of customers in the service system
P_0 = probability that there are zero customers in the service system
P_n = probability that n customers are in the system, such that $n \geq 1$
ρ = average utilization of the system
L_q = the average number of customers waiting in line
W = average time spent in the system (including service)
W_q = average time waiting in line

3.2.1 Single Server Model

A system with one server is known as a single-server model. Recall the equations for the operating characteristics of a single-server model [1]:

$$\begin{aligned} \rho &= \frac{\lambda}{\mu} \\ P_n &= (1-\rho)\rho^n,\ n \geq 0 \\ L &= \frac{\lambda}{\mu - \lambda} \\ L_q &= \rho L \\ W &= \frac{1}{\mu - \lambda} \\ W_q &= \rho W \end{aligned}$$

The parameter ρ is called the "utilization" of the system and gives the probability that the server is busy.

3.2.2 Multi-Server Model

A system with multiple servers is known as a multi-server model. Recall the equations for the operating characteristics for a multi-server queuing model [2]:

$$\begin{aligned} \rho &= \frac{\lambda}{s\mu} \\ P_0 &= \left[\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!}\left(\frac{1}{1-\rho}\right)\right]^{-1} \\ P_n &= \begin{cases} \frac{(\lambda/\mu)^n}{n!} P_0 & 0 < n < s \\ \frac{(\lambda/\mu)^n}{s! s^{n-s}} P_0 & n \geq s \end{cases} \\ L_q &= \frac{P_0 (\lambda/\mu)^s \rho}{s!(1-\rho)^2} \\ W_q &= \frac{L_q}{\lambda} \\ W &= W_q + \frac{1}{\mu} \\ L &= \lambda W \end{aligned}$$

Chapter 4

Queuing at Mountview Middle School

4.1 Assumptions

The parent drop-off situation at Mountview can be viewed as a queuing model where the school is the service system. The customers are the parents, who arrive randomly according to a Poisson distribution. The parents enter the school's driveway and form a single-channel queue behind the drop-off area. The population size is effectively unlimited and the parents are assumed to be patient, waiting to drop off their children on a FIFO basis. We can try to model the flow of cars at Mountview using single-server model and multi-server models. Since it is unclear how many servers (designated drop-off spots) there are, we will apply these assumptions to both single-server and multi-server models.

Under a single-server model, we assume that one parent is dropping off at a time and the drop-off area is the server. Under a multi-server model, we approximate the number of parents that drop off at a time and assume that each drop-off spot is a server. The drop-off area is one lane, so we assumed that drop-off spots in the multi-server models can only serve as many parents as the previous drop-off spot(s). For example, the black car in Figure 4.1 is in drop-off spot 1. If this car is taking a long time to unload their child, the drop-off spots in front of it will not be utilized.

We can collect real-world data to find the values for μ, λ, and s. Once we have these values, we can find the rest of the operating characteristics and represent the drop-off situation at Mountview using different server models.

Figure 4.1: Single-Lane Drop-Off Process

4.2 Methodology for Collecting Data

In order to use queuing theory to represent the situation at Mountview, we visited the school for a total of four days to collect arrival rates, service rates, and data on the number of servers.

4.2.1 Arrival Rates

To collect arrival rates, we visited the school and counted the number of vehicles that arrived between 7:30 a.m. and 8:15 a.m. in five minutes intervals. We denoted whether each vehicle was a parent, bus, or employee. Since arrival rates are dependent on the weather (e.g., if it is raining, kids will be less likely to walk and more parents will be driving their kids to school), we collected arrival rates on three days with no precipitation and on one rainy day. Please see Appendix A for the arrival rate data.

4.2.2 Service Rates and the Number of Servers

To collect service rates for parent drop-offs, we observed the drop-off scenario between 7:30 a.m. and 8:15 a.m. We noticed that cars tend to empty in small groups, so we counted the number of cars dropping off at a time and recorded the amount of time it took the batch of cars to complete their drop-offs. For example, if one car was dropping off, we started the timer once the car parked and stopped the timer when the car started to drive away. If more than one car was dropping off at a time, we started the timer when the first car parked and stopped the timer when the last car in the batch started to drive away. Since service rates are dependent on weather (e.g., if it is raining, there are more parents arriving at the school and there may be larger batches of parents dropping off a time), we collected data on three days with no precipitation and on one rainy day. Please see Appendix A for service rate data.

4.3 Models

Using the data we collected and our assumptions, we created a MATLAB queuing model. To get our average arrival rates, we averaged arrivals during the interval from 7:50 to 8:05a.m., because this is when parent arrivals are the highest at MTV. To get our average service times, we averaged service times for each observed number of parents dropping off at a time. Our model generates a random number of arriving parents according to a Poisson distribution with our average arrival rate, and generates a random number of parents that were served according to an exponential distribution with our average service rate. Based on the balance between the arriving parents and the parents that are served, a queue forms.

Table 4.3 contains data based on a day with good weather. On a sunny day, more children are walking and thus parent arrival rates are lower. Table 4.3 contains data based on a rainy day. On a rainy day, children who usually walk to school get rides from their parents. As a result, parent arrival rates are higher on these days. The column called, ‘Data’ contains data that we collected at Mountview. The columns called, ‘Single-Server,’ ‘2 Servers,’ and ‘3 Servers’ contain data that we generated with the MATLAB model. We ran each model 10 times and averaged the results from the 10 simulations.

	Data	**Single-Server**	**2 Servers**	**3 Servers**
# arriving parents per min	4.3	4.1	4.3	3.9
# drop-off spots (channels)	2.6	1	2	3
Queue	2.7	7.9	5.6	5.5

Table 4.1: Good Weather Day

Not all of the data matches up perfectly. For example, the average queue length for a single-server model on a sunny day is much higher than our

	Data	**Single-Server**	**2 Servers**	**3 Servers**
# arriving parents per min	6.1	5.5	5.6	5.5
# drop-off spots (channels)	2.8	1	2	3
Queue	8.2	12.6	9.5	6.8

Table 4.2: Bad Weather Day

observed value. This difference could be because in a single-server model, there is only one parent dropping off at a time. In reality, there may be two, three, or even more parents dropping at once. The average model queue lengths for a day with good weather are all higher than our observed value. This difference could be due to the fact that these models may not be suitable for describing the situation, or our observed data may be faulty. We may have only noticed the queue when it was short, creating a bias in our data collection.

The average queue length for a single-server model on a rainy day is higher than our observed value. This difference could be because in a single-server model, there is only one parent dropping off at a time. In reality, there may be two, three, or even more parents dropping at once. The average queue length for the three-server model is lower than our observed value. This difference could be because in a three-server model, there are always three cars dropping off at a time. In reality, there may be only one car dropping off at certain points.

Chapter 5

Optimization

Mountview Middle School is not unique; many schools have faced excessive traffic congestion that makes morning drop-offs a headache for all parties involved. This section discusses various solutions, including past recommendations made to schools with similar traffic problems and solutions that we brainstormed for the unique problem at Mountview Middle School.

5.1 Verbal Form of the Optimization Problem

The overall objectives for alleviating the traffic congestion at Mountview are to prevent spillover onto Shrewsbury Street, reduce average queue lengths, and reduce time parents spend in the system. There are various people who contribute to or are affected by the traffic problem caused by the drop-off situation at Mountview Middle School. These constituents include:

- Parents
- Kids
- Principal and teachers at Mountview Middle School
- Commuters
- Bus drivers

Each constituent has their own optimization problem that needs to be solved. Parents want to drop off their kids as quickly as possible so they can return home or head off to work. They want their time in the system to be minimized, and to accomplish this, their time waiting in the queue and their service time need to be minimized. The parents want their child to be safe, but do not necessarily show consideration for the safety of the other children.

Kids want to feel safe and get to school on time to avoid being disciplined by their teachers. To ensure the safety of the kids, there needs to be little confusion during the drop-off process. The drop-off rules and regulations need to be clearly spelled out and should be clearly explained to the parents so they can follow them strictly. To make sure the kids get to school on time, the parents need to be able to drop off the kids quickly and efficiently. If the optimization problem for the parents is solved, then the children will be on time for school.

The staff at Mountview Middle School want the children to be safe, but they also want minimal complaints from parents. Employees also need an adequate number of parking spaces and easy access to them. If the optimization problems for the parents and the children are satisfied, then the optimization problem for the staff at Mountview will also be satisfied.

Time is money, so the commuters want to get to work on time and avoid traffic back-ups along the way. If there are a lot of parents waiting to drop off their kids and they fill up the queue in the school driveway, the queue will spill out onto Shrewsbury Street. If the commuters can't drive around the queue, then they will end up waiting behind the parents. To prevent this traffic, the parents need to be able to get onto the school site quickly and be able to drop off quickly enough to prevent a large queue. If the optimization problem for the parents is satisfied, then the commuters will also be content.

The bus drivers' job is to get the kids to school on time and to protect these children. Traffic back-up and confusion during the drop-off process should be minimized. Buses are wide vehicles and need road space that is unobstructed in order to make sharp turns.

Overall, if the optimization problem for the parents can be solved, then the remaining constituents will be content. If parents get through the system quickly and do not form large lines behind the drop-off area, then there will be little to no spillover onto Shrewsbury Street. Parents and commuters will be happy, children will be on time, and as a result, employees of MTV will also be happy.

5.2 Solutions: Changing the Parameters

There are a number of ways the optimization problem can be approached. Simply from an abstract queuing perspective, we can look at changing the various parameters of the system: the maximum size of the queue, the arrival rate of parents, the service time of the parents dropping off their kids, and the number of servers. Table 5.2 lists the parameters of the system based

Discrete	**Continuous**
Number of drop-off spots for parents	Service time
Arrival rate	Length of queuing area

Table 5.1: Discrete vs. Continuous Parameters

on whether they are discrete or continuous. By changing the parameters, we hope to change the behavior of the system in order to decrease the queue lengths and help parents move through the system faster. We brainstormed various ideas with little regard to feasibility or price in order to have a thorough set of possible solutions.

Changing the Number of Drop-Off Spots

To increase the number of parents that can drop off at a time, we need to increase the number of drop off spots. At Mountview, we recorded how many parents were dropping off at a time and the number of cars queuing up at that particular moment. We averaged the queue lengths for one, two, and three parents dropping off at time and then plotted the relationship between the number of servers and the average queue length. We took a linear regression (see Figure 5.1) and found that the data has a coefficient of determination (R^2) value of .83, so we can conclude that the relationship is nearly linear. In other words, as we increase the number of servers, the average queue of waiting parents decreases. However, we must keep in mind that even if there are a lot of parents dropping off at once, parents might still spend a lot of time in the system if the service times are very high. Additionally, increasing the number of drop off spots has safety repercussions because it usually adds confusion to the drop-off process. Ideally, there should be an adequate number of servers and a reasonable service time to avoid high traffic volumes.

- *Paint lines distinguishing drop-off spots*

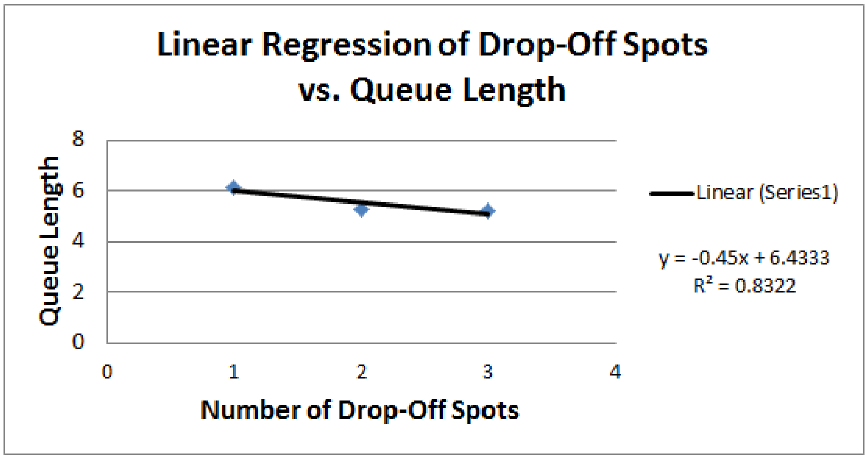

Figure 5.1: Relationship Between the Number of Drop-Off Spots and Average Queue Length

Currently, there are no lines marking off areas to drop off. Parents do not know how far to pull up, and as a result there are varying number of parents dropping off at once. If we paint lines distinguishing drop-off spots, we can decrease the confusion during the drop-off process and potentially have more parents dropping off at once.

- *Move the exit lane further east*

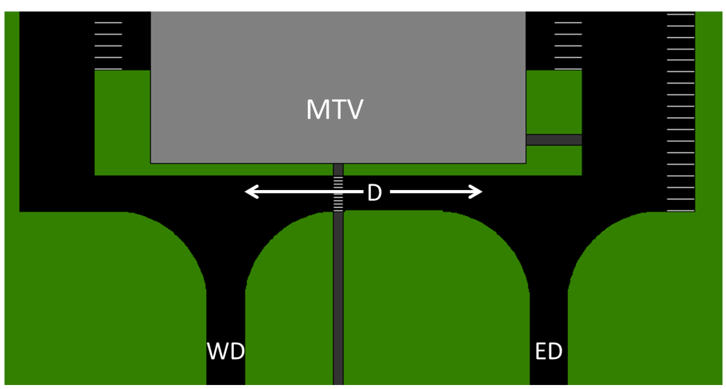

Figure 5.2: Extended Drop-Off Area

By moving the exit lane to the east, we can extend the drop-off area and have more parents dropping off at once (see Figure 5.2).

- *Create two drop-off lanes*

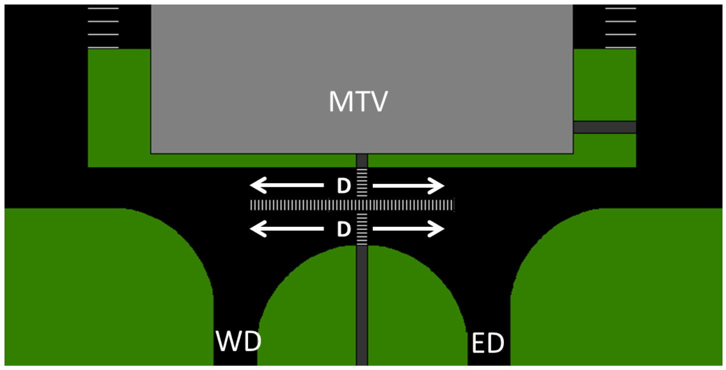

Figure 5.3: Two Lane Drop-Off

If we shorten the green, we can widen the drop-off area enough to make room for two drop-off lanes. The drop-off lanes would be separated by a crosswalk so children dropped off in the lane further away from the school will have a safe place to walk. The two lanes would, under perfect circumstances, double the number of parents who could drop off at a time (see Figure 5.3).

Changing the Arrival Rate

Currently, there is a high number of parents who arrive between 7:50 and 8:10 a.m. due to the morning rush. If we can somehow decrease the number of parents who drive their kids to school or spread out the number of parents arriving over a larger time interval, we can prevent high amounts of traffic congestion [8].

- *Encouraging children to walk, bike, or take the bus*
- *Encouraging parents to carpool*
- *Add a bus route that strictly picks up students who live within walking distance of the school*

  Children that live within a certain proximity to the school are not allowed to ride the bus to school; if we allow buses to pick up children that live within this proximity, we could decrease the number of parents driving their kids to school.

- *Stagger the drop-offs*

  We could assign parents to a particular time frame in which they have to drop off their children. For example, we could assign time frames based on proximity to the school or grade level. In this way, we could prevent arrivals from being highly concentrated in one time period.

- *Expand the time frame for dropping off*

  Currently, parents are not allowed to drop off their children before 7:30 a.m. If we change this time to an earlier time, the number of parents dropping off their children during the busiest time might decrease.

- *Charge parents for the ability to drive their kids to school*

Changing the Service Time

One way to get parents through the system faster is to decrease the time it takes them to drop off their children.

- *Hire someone to direct the drop-off process*

  Occasionally, one of the teachers will direct the drop-off process by standing in the drop-off area and telling cars where to pull up to. This strategy always results in an efficient drop-off process. If the school hired someone or required teachers to take turns directing the traffic, the traffic congestion could be alleviated significantly.

- *Switch the direction of traffic flow*

  Parents currently drop off their children with the driver-side of their vehicle facing the building. Oftentimes, children are sitting in the passenger seat, and once they exit the vehicle they have to walk in front of the vehicle before walking into the school. If we switched direction of the traffic flow so vehicles enter the East Driveway and exit the West Driveway, parents would drop off with the passenger-side of their vehicle facing the school. This solution would prevent children from walking in front of vehicles (maximizing safety [7]) as well as decrease the service time (see Figure 5.4).

- *Switch the direction of parent traffic flow, but keep the flow of bus and employee traffic the same*

  Parents will be able to drop off their kids with the passenger side of the car facing the building, potentially decreasing the service time. At the same time, buses will have adequate space to queue up next to the gym entrance (see Figure 5.5).

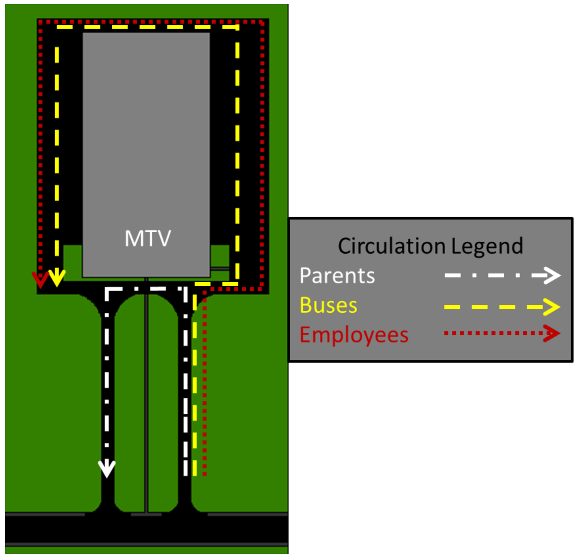

Figure 5.4: Switching the Direction of Traffic

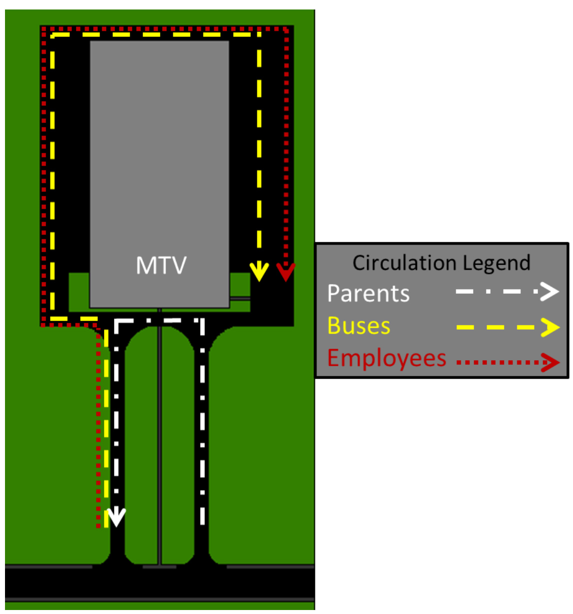

Figure 5.5: Switching the Direction of Parent Traffic

- *Add a passing lane*

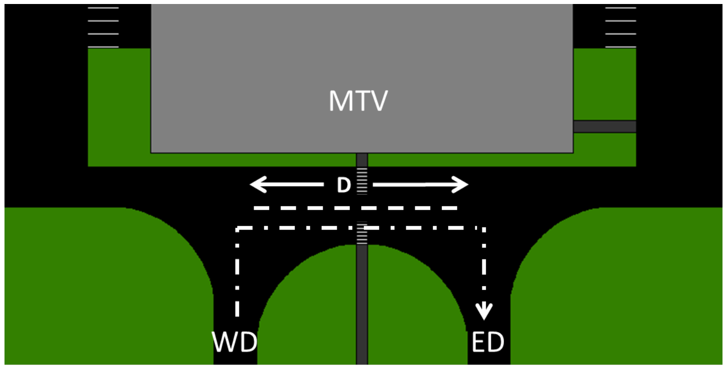

Figure 5.6: Passing Lane

We could shorten the green and make the drop-off area have two lanes, one for dropping off and one for passing (see Figure 5.6). Adding a passing lane would eliminate our previous assumption that each drop-off spot can only serve as many parents as drop-off spots before it. By eliminating this assumption, we know that each drop-off spot can serve approximately the same number of parents, giving us a standard queuing model. We can use the operating characteristic for a standard queuing model to predict the average queue length when the school has a passing lane. In Table 5.2, we took rainy day arrival and service rates and compared the average queue length we observed with queue lengths for standard multi-server models. Although the queue lengths we calculated would only appear under perfect circumstances, the change is still significant.

Bad Weather	Observed Data	2 Servers	3 Servers
Queue Length	8.2	2.3	.7

Table 5.2: Average Queue Lengths with a Passing Lane

Changing the Length of the Queuing Area

- *Create a left-turn lane on the eastbound side of Shrewsbury Street*

  If there is a spill over of parents onto Shrewsbury Street, a left-turn lane will allow parents to queue up and give commuters the ability to easily pass by the school.

- *Switch the drop-off areas for parents and buses*

  If parents circled around the building and dropped off in front of the cafeteria entrance, there would be a much larger area for them to line up. This increase in queuing area would prevent cars from spilling over onto Shrewsbury Street.

- *Make an entrance lane off of a side road*

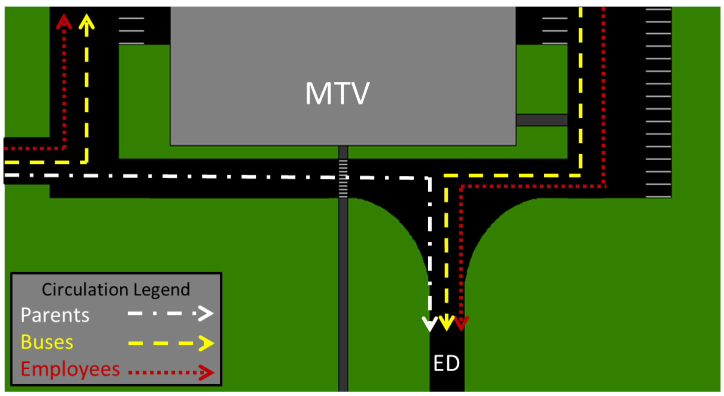

Figure 5.7: Entrance on Side Road, One Exit

  In this solution, we would create an entrance lane off of a side road west of the school and remove the original entrance lane so there is one exit lane onto Shrewsbury Street. With a new entrance lane, the area for queuing would be extended and parents would not spill over onto Shrewsbury Street (see Figure 5.7).

- *Move the school building 200 yards back on the school's property*

  We can increase the length of the driveways, and consequently, the area for queuing, by moving the school 200 yards back on the property.

Changing Service Time and the Number of Servers

- *Relocate the pedestrian walkway*

  If a child approaches the crosswalk when parents are in the midst of dropping off their children, it takes longer for parents to pull out of the drop-off area. If a child starts to use the crosswalk when the drop-off area is empty and parents approach the area, the parents will stop at the edge of the crosswalk to unload. Since only one or two cars usually

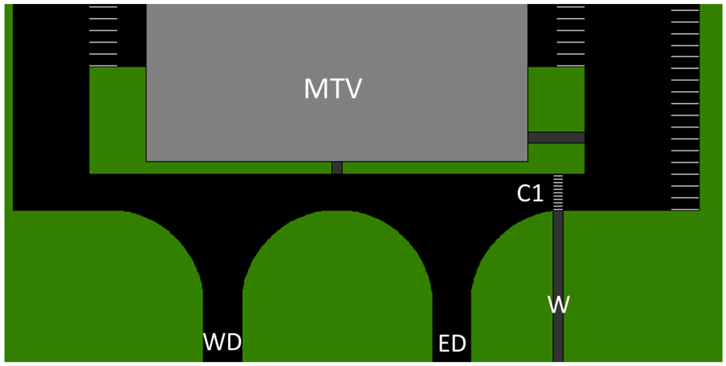

Figure 5.8: Relocating the Pedestrian Walkway

drop off before the crosswalk, a child in the crosswalk decreases the potential number of parents that could be dropping off at once. If we relocate the pedestrian walkway to the grassy area east of the school's driveways, we can increase the number of drop-off spots and decrease the service time (see Figure 5.8).

- *Have parents circle the building and drop off in the exit lane*

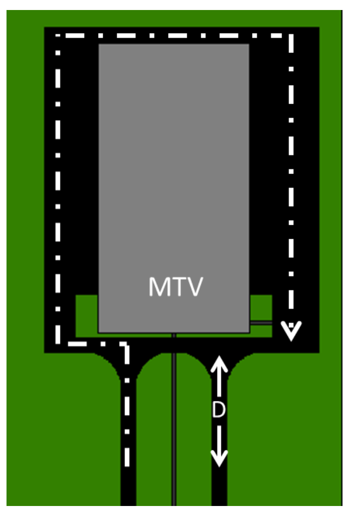

Figure 5.9: Move Parent Drop-Off Area to Exit Lane

If we require parents to circle the building before dropping off in the exit lane, they can potentially decrease their service time by avoiding driving through the crosswalk. Since the drop-off area is now in one of the exit lanes, there is more room for more parents to drop off at once (see Figure 5.9).

- *Add an exit driveway*

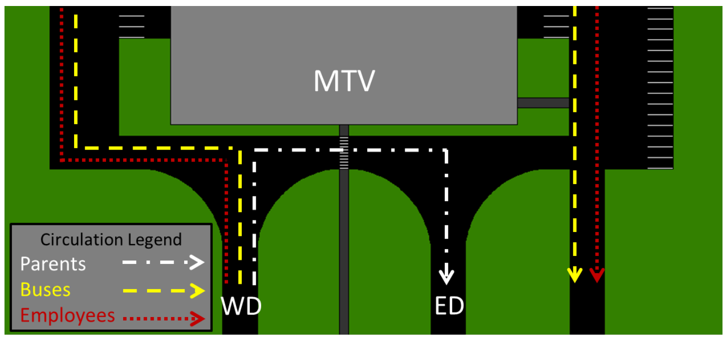

Figure 5.10: Extra Exit Driveway

This solution involves adding an exit driveway, which will allow parents to have one exit and buses and employees to have the other exit (see Figure 5.10). The drop-off area for parents could be extended into the exit driveway for parents. One lane in the parents' exit driveway can be used to drop off kids and one lane can be used as a passing lane. More parents can drop off at a time, and the passing lane will allow parents to leave the system who otherwise would be stuck behind slower vehicles. The passing lane would decrease the average service time for parents.

- *Add an entrance driveway*

This solution involves adding an entrance driveway so there is one entrance for parents and one for buses and employees. In the parents' entrance driveway, one lane can be used to drop off kids and one lane can be used as a passing lane. More parents will be able to drop off at a time, and the passing lane will allow parents to leave the system who otherwise would be stuck behind slower vehicles. In other words, the passing lane would decrease the average service time for parents.

- *Require parents to drop off and then circle the building*

Requiring parents to drop off and then circle the building would prevent them from driving through the crosswalk.

Changing the Length of the Queuing Area,

Number of Servers, and Service Time

- *Create an entrance lane off of a side road with one exit driveway onto Shrewsbury Street*

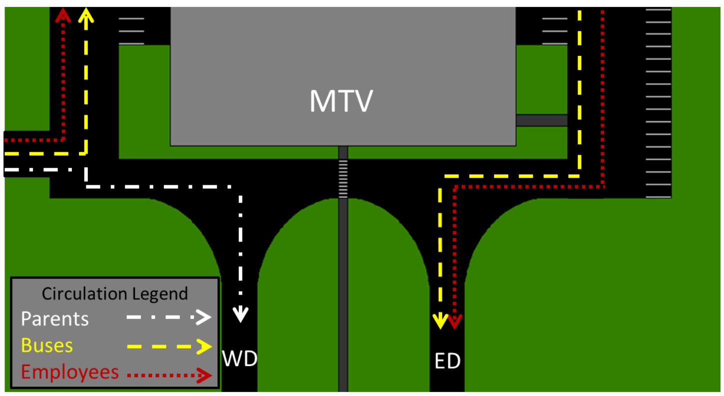

Figure 5.11: Entrance Off Side Road, Two Exits

In this solution, we are creating an entrance lane off of a side road west of the school and removing the original entrance so there is one exit lane onto Shrewsbury Street (see Figure 5.7).The new entrance lane would increase the queuing area and prevent spillover onto Shrewsbury Street. The drop-off area for parents could be extended into the exit driveway for parents. One lane in the exit driveway for parents can can be used to drop off kids and one lane can be used as a passing lane. The passing lane will allow parents to leave the system who otherwise would be stuck behind slower vehicles.The passing lane would decrease the average service time for parents.

Chapter 6

Markov Chains: an Alternative to Queuing Theory

A discrete-time stochastic process is a series of events where the outcome of any event depends on a certain probability [6]. In a Markov process, the events are a stochastic process that have several additional properties:

- The number of possible states is finite.
- The state at time $t + 1$ only depends on the state at time t and some purely random numbers.
- The probabilities of each event (i.e. state-to-state transition) are constant over time.

The likelihood of each state-to-state transition in a Markov process can be represented by a matrix called a **transition matrix** [3]. If there are n states in the Markov process, the transition matrix T is an nxn matrix where the $(i, j)^{\text{th}}$ entry represents the probability that an object in state j will enter state i after one time period.

We can look at the drop-off process for parents at Mountview as a Markov process. One possible transition matrix can be created by assuming that there are five states for parents during the drop-off process:

- State 1: Entering the school premises
- State 2: In the queue

- State 3: Dropping off their child
- State 4: Waiting to leave the school grounds
- State 5: Not on school grounds

We assign the following probabilities to each possible state-to-state transition:

$$p_i = \text{probability that a parent will stay in state } i$$
$$1 - p_i = \text{probability that a parent will move on to the next state}$$

Using the probabilities of each state-to-state transition, we can create a transition matrix for the drop-off process. The transition matrix T for the drop-off process at MTV is shown below.

$$T = \begin{pmatrix} p_1 & 0 & 0 & 0 & 1-p_5 \\ 1-p_1 & p_2 & 0 & 0 & 0 \\ 0 & 1-p_2 & p_3 & 0 & 0 \\ 0 & 0 & 1-p_3 & p_4 & 0 \\ 0 & 0 & 0 & 1-p_4 & p_5 \end{pmatrix}$$

If we know the initial number of parents at each stage of the process, we can create a vector, x_0, which represents the initial state of the system [6]. When we multiply the transition matrix T by x_0, we get the state of the system after one time period. If we keep multiplying the initial state vector by T, we get a chain of vectors:

$$x_0, Tx_0, T^2x_0, ..., T^nx_0$$

where T^nx_0 represents the system after n time periods. Eventually, the proportion of parents at each stage of the process will cease to change; in other words, the state of the system will approach a steady-state x_s:

$$\lim_{n\to\infty} T^nx_0 = x_s$$

This steady state is true provided that x_0 is not in some other eigenspace. So $Tx_s = x_s$ and we have an eigenvector with an eigenvalue equal to 1.

If we know the initial state of the system and the probabilities for the transition matrix, we can represent the drop-off situation at Mountview as a Markov chain. This transition matrix allows us to see which states have the largest proportion of parents at large time.

Chapter 7

Conclusion

If we can understand the traffic dynamics at MTV, then we can begin to understand traffic problems of greater magnitude. We modeled the parent drop-off process using single-server and multi-server queuing models. We created a simulation in MATLAB that illustrates the queue of parents waiting to drop off their children. The simulation also shows how large queues result in a spillover of cars onto Shrewsbury Street. We also explored Markov Chains as an alternative model to represent the drop-off process. We then explored the various parameters that we can change to improve the drop-off process, and discussed the various solutions for redesigning the drop-off area at the school that could potentially alleviate some of the traffic congestion.

7.1 Future Work

One possible topic for future research related to this project is to investigate the plans for the school's new building. The town of Holden recently approved the plan to build a new building for Mountview Middle School (see Figure 7.1 for a picture of the new school building). The new building will be located about 200 yards back on the school's property, which will result in longer driveways. The parents will drop off in the back of the school, and the buses will queue up in front of the school. The flow of traffic will also be the opposite of the current flow; all vehicles will enter through the East Driveway (the driveway on the right) and exit through the West Driveway (the driveway on the left). The new flow of traffic will match standard traffic conventions. Future researchers could apply a queuing model (or another operations research model) to the design of the new school to see if the design of the drop-off process reduces service time for parents and prevents queues from spilling onto Shrewsbury Street. Since the school will not be completed for several years, there will be an increase in traffic resulting from the construction process. A future researcher could design solutions to alleviate some of the traffic congestion during the construction of the new building.

Figure 7.1: Mountview's New School Design

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Appendix A

Data Collection

Table A.1: Day 1 Arrivals

Time	Buses	Employees	Parents	Total
7:30-7:35	0	5	10	15
7:35-7:40	0	2	7	9
7:40-7:45	0	6	8	14
7:45-7:50	2	5	9	16
7:50-7:55	5	7	17	29
7:55-8:00	4	5	18	27
8:00-8:05	3	5	20	28
8:05-8:10	0	2	21	23
8:10-8:15	0	1	11	12
Totals	14	38	121	173

Table A.2: Day 2 Arrivals

Time	Buses	Employees	Parents	Total
7:30-7:35	0	6	4	10
7:35-7:40	0	6	6	12
7:40-7:45	0	5	8	13
7:45-7:50	1	5	10	16
7:50-7:55	6	1	12	19
7:55-8:00	6	10	23	39
8:00-8:05	3	4	20	27
8:05-8:10	0	0	21	21
8:10-8:15	0	0	4	4
Totals	16	37	108	161

Table A.3: Day 3 Arrivals

Time	Buses	Employees	Parents	Total
7:30-7:35	0	9	14	23
7:35-7:40	0	3	7	10
7:40-7:45	0	3	5	8
7:45-7:50	1	4	7	12
7:50-7:55	6	7	10	23
7:55-8:00	6	7	30	43
8:00-8:05	1	4	21	26
8:05-8:10	0	2	19	21
8:10-8:15	0	1	6	7
Totals	14	40	119	173

Table A.4: Average Parent Arrivals (per minute)

Time	Day 1	Day 2	Day 3	Average
7:30-7:35	2.00	0.80	2.80	1.87
7:35-7:40	1.40	1.20	1.40	1.33
7:40-7:45	1.60	1.60	1.00	1.40
7:45-7:50	1.80	2.00	1.40	1.73
7:50-7:55	3.40	2.40	2.00	2.60
7:55-8:00	3.60	4.60	6.00	4.73
8:00-8:05	4.00	4.00	4.20	4.07
8:05-8:10	4.20	4.20	3.80	4.07
8:10-8:15	2.20	0.80	1.20	1.40

Table A.5: Day 1 Drop-Off Times

Time		Service Times									
7:30-7:35	No. of cars	1	4	2	1	1	1	1	1		
	Service Times (s)	16	18	15	18	15	30	30	17		
7:35-7:40	No. of cars	1	1	2	1	2					
	Service Times (s)	20	35	17	12	23					
7:40-7:45	No. of cars	1	1	2	1	1					
	Service Times (s)	28	11	31	9	11					
7:45-7:50	No. of cars	2	1	1	1	1	1	1			
	Service Times (s)	18	15	11	8	25	9	12			
7:50-7:55	No. of cars	1	1	2	2	3	1	2	2	1	1
	Service Times (s)	14	16	35	18	24	26	20	35	21	10
7:55-8:00	No. of cars	4	1	2	3	2	3	2			
	Service Times (s)	35	10	29	24	15	40	20			
8:00-8:05	No. of cars	3	2	3	3	2	1	2	3	3	
	Service Times (s)	17	15	15	27	10	11	16	31	28	
8:05-8:10	No. of cars	4	2	1	1	4	2	4			
	Service Times (s)	38	25	10	15	23	23	32			
8:10-8:15	No. of cars	2	2	1	1	1	1	3			
	Service Times (s)	18	14	8	12	15	13	22			

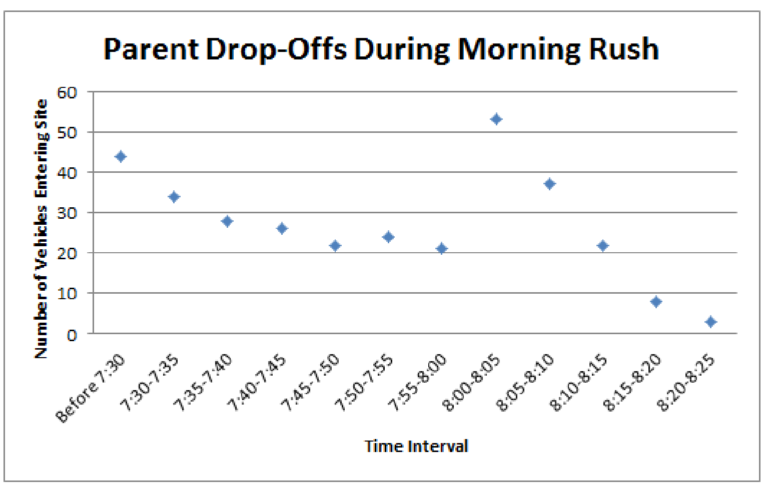

Figure A.1: Data Collected by Nitsch Engineering

Table A.6: Day 2 Drop-Off Times

Time		Service Times									
7:30-7:35	No. of cars	1	1	1	1						
	Service Times (s)	16	8	15	10						
7:35-7:40	No. of cars	1	3	1	1	1					
	Service Times (s)	14	39	5	8	25					
7:40-7:45	No. of cars	1	1	1	2	1	1				
	Service Times (s)	14	25	13	23	22	10				
7:45-7:50	No. of cars	3	2	1	1	2					
	Service Times (s)	46	17	19	8	39					
7:50-7:55	No. of cars	2	3	1	2	3	1				
	Service Times (s)	30	23	28	20	17	12				
7:55-8:00	No. of cars	1	3	2	2	1	2	4	4	3	3
	Service Times (s)	10	37	23	8	11	36	39	23	18	17
8:00-8:05	No. of cars	3	3	4	3	2	2	1	2		
	Service Times (s)	27	15	23	35	17	31	17	8		
8:05-8:10	No. of cars	1	3	3	3	2	4	3	1		
	Service Times (s)	9	33	20	18	24	40	12	25		
8:10-8:15	No. of cars	1	1	1							
	Service Times (s)	6	14	23							

Table A.7: Day 3 Drop-Off Times

Time		Service Times								
7:30-7:35	No. of cars	1	3	2	1	1	1			
	Service Times (s)	13	26	23	4	7	15			
7:35-7:40	No. of cars	2	1	2	1					
	Service Times (s)	28	17	29	16					
7:40-7:45	No. of cars	3	1	1						
	Service Times (s)	21	9	22						
7:45-7:50	No. of cars	1	2	2	1					
	Service Times (s)	13	30	15	17					
7:50-7:55	No. of cars	1	1	3	4	3	3			
	Service Times (s)	18	23	52	38	22	20			
7:55-8:00	No. of cars	3	4	4	3	3	4			
	Service Times (s)	20	30	17	23	30	40			
8:00-8:05	No. of cars	3	3	2	2	4	4	3	2	4
	Service Times (s)	24	20	16	12	30	35	22	17	30
8:05-8:10	No. of cars	2	3	2	1	1	1	1		
	Service Times (s)	13	25	14	8	8	10	10		
8:10-8:15	No. of cars	1	1	1	1	1	1			
	Service Times (s)	7	8	8	10	5	8			

Table A.8: Arrivals

Time	Buses	Employees	Parents	Total
7:30-7:35	0	5	10	15
7:35-7:40	0	5	21	26
7:40-7:45	0	2	12	14
7:45-7:50	4	2	15	21
7:50-7:55	4	6	24	34
7:55-8:00	5	10	29	44
8:00-8:05	1	5	35	41
8:05-8:10	0	1	27	28
8:10-8:15	0	0	6	6
Totals	14	36	179	229

Table A.9: Average Arrivals of Parents

Time	Arrivals (per minute)
7:30-7:35	2
7:35-7:40	4.2
7:40-7:45	2.4
7:45-7:50	3
7:50-7:55	4.8
7:55-8:00	5.8
8:00-8:05	7
8:05-8:10	5.4
8:10-8:15	1.2

Table A.10: Rainy Day Drop-Off Times

Time	Service Times														
7:30-7:35	No. of cars	1	1	1	2	1	1	1	2						
	Svc Time(s)	17	10	8	30	21	8	12	25						
7:35-7:40	No. of cars	3	2	1	3	3	2	1	2						
	Svc Time(s)	23	22	11	26	24	8	12	22						
7:40-7:45	No. of cars	2	3	1	1	1	1	2	2	1	1				
	Svc Time(s)	20	25	10	10	8	17	20	16	12	18				
7:45-7:50	No. of cars	1	2	2	3	2	2	2							
	Svc Time(s)	11	30	16	30	14	8	20							
7:50-7:55	No. of cars	2	1	2	3	1	1	1	2	4	3	2			
	Svc Time(s)	18	13	17	30	19	14	25	30	22	22	19			
7:55-8:00	No. of cars	4	2	3	3	3	3	2	1	2	3	1	3	4	
	Svc Time(s)	42	13	20	18	20	14	23	10	8	14	5	34	20	
8:00-8:05	No. of cars	3	4	2	4	4	3	5	3						
	Svc Time(s)	21	30	13	18	18	12	20	18						
8:05-8:10	No. of cars	3	3	1	1	4	2	3	3	1	3	3	1	2	1
	Svc Time(s)	15	24	11	9	28	17	13	11	8	22	19	43	19	15
8:10-8:15	No. of cars														
	Svc Time(s)														