Portfolio Optimization Based on Robust Estimation Procedures

A Thesis

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by

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1. Introduction

The goal of the portfolio manager is to invest in securities that offer returns superior to the returns on a benchmark at an acceptable risk. Given the large number of available securities, this selection process has to be designed in a systematic fashion, so that it allows to be implemented with the help of computers.

The state-of-the art methodology for portfolio selection is the use of mathematical optimization. Typically one would minimize a quadratic objective function interpreted as risk, subject to constraints representing the minimal required return and various policies affecting the portfolio construction. Examples of such policies are limitation on the maximum position the manager can take in a single security, restriction on short sales or the requirement to be fully invested. The solution gives the allocations to be invested in the various available securities.

The optimization program requires as input parameters, the coefficients of the objective and constraint function that express the risk and the expected return for every security. In the practice these parameters are not known and have to be estimated from observed market returns. The challenge arises from the fact that as market condition shift, the risk and expected return of the various securities change. As a result, the input parameters of the optimization program have to be re-estimated periodically and the portfolio composition has to be rebalanced. The estimation of the parameters can only based on observed market data that are still relevant for the period when the rebalanced portfolio is going to be invested. Due to the constantly shifting market conditions, only a relatively short period of market history can be used for the estimation.

The portfolio problem depends on a large number of parameters that need to be estimated on the basis of relatively small samples. In such cases a few outliers can have a significant negative effect on the statistical estimates and ultimately on the performance of the optimized portfolio based on them.

The traditional statistical estimates are based on the least squares method, which has ideal theoretical properties for large samples and for normally distributed variables. As outlined above, the portfolio manager cannot rely on estimates based on large samples. Moreover it is empirically known that distributions of financial returns have fatter tails than the normal distribution resulting in more outliers. Least squares estimators are very sensitive to outliers, especially if the sample size is not large enough.

The objective of the present thesis is to investigate the use of more robust estimation procedures in conjunction with portfolio optimization.

To reduce the number of parameters that need to be estimated we consider the widely used single-index model originally proposed by Sharpe [1] We estimate the necessary parameters both by the traditional ordinary least squares regression method and by the recently developed robust regression by Huber [2], Yohai and Zamar[3]. We feed the output of the two estimation procedures into the portfolio optimization program and compare the performance of the resulting portfolios.

We perform our comparative analysis on a subset of small capitalization stocks from the Russell2000 index, which have sufficiently large daily trading volumes to qualify for inclusion in an institutional investment portfolio. We use the Russell 2000 index as the benchmark. The reason for our focus on small capitalization stocks is twofold. They offer higher returns in exchange for higher risk and, from the statistical point of view they give rise to a higher number of outliers than the more stable and predictable large companies. Also the mathematical-computational approach is more important for small stocks, for which very little fundamental research is available making traditional investment analysis more difficult.

We compare the performance of two optimized portfolios to an equal weighted passive portfolio and to the Russell 2000 index over the 16 business quarters between Jan 1, 2000 and Dec 31, 2003. This interval covers both good and difficult, turbulent periods of US financial markets. The two optimized portfolios are rebalanced quarterly. Trading costs resulting from the rebalancing are subtracted from the portfolio values. The parameters needed for the optimizer are based on price data from preceding quarters. One of the portfolios is using the ordinary least squares regression estimates the other one is based on robust regression.

2. Background

2.1 The minimum risk, required return portfolio problem

A Portfolio is a collection of n securities. The return on the portfolio R_p is a weighted sum of the returns on the individual securities:

$$R_p = \sum_{i=1}^{n} x_i R_i \tag{1.1}$$

Where x_i is the investment in the security i. The daily returns R_i on security i is a random variable. They are assume to be independent between different days and correlated among the various securities on the same day. The expected daily return a_i of security i and the covariance σ_{ij} between two securities are defined as:

$$a_i = E\,R_i, \tag{1.2}$$

$$\text{and } \sigma_{ij} = E(R_i - a_i)(R_j - a_j) \tag{1.3}$$

Using this notation, the expected return of the portfolio a_p and the variance of the daily returns of the portfolio can be expressed as:

$$a_p = \sum_{i=1}^{n} x_i a_i = X^T a \tag{1.4}$$

$$\text{and } \sigma_p^2 = \sum_{j=1}^{n}\sum_{i=1}^{n} x_i \sigma_{ij} x_j = X^T \Sigma X \tag{1.5}$$

Here we used the vector, matrix notation:

$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \; and \; \Sigma = \begin{bmatrix} \sigma_1{}^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \sigma_{n1} & \cdots & \cdots & \sigma_n^2 \end{bmatrix}$$

The variance of the portfolio returns is considered the portfolio risk. The objective of the minimum risk, required return portfolio problem is to find portfolio weights

$X = (x_1, x_2, \cdots, x_n)^T$ to

Minimize $X^T \Sigma X$

Subject to $X^T a \geq a_{required}$ (1.6)

And possible to some more constraints on the weights $X = (x_1, x_2, \cdots, x_n)^T$

Typical addition constraints are:

$$\sum_{i=1}^{n} x_i = P \tag{1.7}$$

meaning that the entire value P must be invested, or

$$0 \le x_i \le zP, i = 1, \cdots, n \tag{1.8}$$

which means that no short positions are allowed and that not more than the fraction $0 \le z \le 1$ of the entire value can be invested in a single security.

Expressions (1.6-1.8) define a (linearly) constrained quadratic optimization problem. Efficient algorithms for the solution of this optimization problem exist, but they require that the input quantities a, Σ (and z, P) are known.

In reality security prices, and hence the security returns R_i are random variables whose realizations can be observed. The expected values a_i and covariances σ_{ij} are not known. They need to be estimated from a statistical sample of daily returns. The quality and hence the usefulness of the results of the portfolio optimization problem critically depend on the quality of the statistical estimates of these input parameters.

In portfolio analysis one is faced with the challenge of two conflicting demands. Good quality statistical estimates require large sample size. A rule of thumb is that the number of observations must be at least as large as the number of the different elements of the covariance matrix. For a portfolio involving 100 securities this would mean observations from 5050 trading days, which is about 20 years. This is obviously not possible. Since market conditions change much more rapidly. Using outdated observations would result in estimates that are irrelevant to current or future market conditions

The next section outlines an approach to significantly reduce the number of parameters that need to be estimated.

2.2 The single index model

The purpose of the single index model is to express the large number of covariances between the individual securities though a significantly smaller number of

$$E(e_i, e_j) = 0$$

parameters. To make this possible we make the crucial assumption that the sole reason for the correlation between two securities is their dependence on a common market index. In other words that there are no direct connections between the movements of the securities, only the indirect connection through the market index. This can be formally expressed in the following form:

$$R_i = \alpha_i + \beta_i R_m + e_i \tag{2.1}$$

Here R_m is the return on the market index and e_i is a zero mean random error term, Specific to security i. The key assumption is that the e_i's are independent of (or uncorrelated with) R_m and all other e_j's with $i \neq j$. Formally, the assumptions can be written as:

$$E\ e_i = 0 \tag{2.2}$$

$$E[e_i(R_m - \overline{R_m})] = 0 \tag{2.3}$$

$$E(e_i e_j) = 0 \tag{2.4}$$

$$\operatorname{var} e_i = Ee_i^2 = \sigma_{ei}^2 \tag{2.5}$$

$$\operatorname{var} R_m = E(R_m - \overline{R}_m)^2 = \sigma_m^2 \tag{2.6}$$

Using the regression model (2.1), elements of the variance-covariance matrix can be expressed in the form:

$$\sigma_i^2 = \sigma_{ii} = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2 \tag{2.7}$$

$$\sigma_{ij} = \beta_i \beta_j \sigma_m^2 \tag{2.8}$$

Under the additional assumptions of the single index model the n(n+1)/2 different elements of the variance-covariance matrix can be expressed by 2n+1 parameters β_I σ_{ei}^2 σ_m^2. This is a significant reduction of the number of parameters with will need to be estimated from samples r_{ij}, r_{mj} of observed security and market returns. Here, j = 1…J runs over the days which are providing the statistical sample of the historical returns.

2.3 Ordinary least squares (OLS) regression method

The traditional way to estimate the regression parameters α_I, β_I and σ_{ei} is the least squares method. This chooses the parameters $\hat{\alpha}_i, \hat{\beta}_i, \hat{\sigma}_{ei}$, which minimize the sum of squared differences between the left and right hand sides of the regression equation(2.1) for the observed sample.

$$\sum_{j=1}^{J} \left[r_{ij} - (\hat{\alpha}_i + \hat{\beta}_i r_{mj}) \right]^2 = \min_{\alpha_i, \beta_i} \sum_{j=1}^{J} \left[r_{ij} - (\alpha_i + \beta_i r_{mj}) \right]^2 \tag{3.1}$$

Under a set of standard assumptions, usually referred as the Gauss-Morkov assumptions, the OLS regression estimates have the desirable properties of unbiasedness and effectiveness. This means that they are the best possible estimates in the sense that their estimated error has mean zero and minimal variance.

Despite of these desirable theoretical properties, ordinary least squares are not ideally suited to estimated the parameters α_i, β_i and σ_{ei} for the financial problem of the single index model.

One of the fundamental Gauss-Morkov assumptions underpinning the OLS method is the requirement that the error terms e_i are normally distributed. This implies that the daily securities returns Ri, conditionally on the index returns R_m, would also be normally distributed. Empirical evidence does not support these assumptions. Outliers are much more frequent in equity return than what would follow from the fast declining tails of the normal distribution. It is, in fact, generally accepted that the distributions of security returns have “fat tails”.

The measure of frequent outliers destroys the desirable statistical properties of the OLS estimates. In practical terms this means that OLS estimates are overly sensitive to outliers. One or tow outsized daily return can have a disproportional strong influence on the alpha or beta estimates making a security to appear much more or much less desirable than it should be.

2.4 The robust regression method

To estimate the regression coefficients in the presence of fat-tailed distributions, new estimation methods need to be developed, that suppressed the outsized effects of the outliers. The key is either to systematically cut out the outliers or to use a penalty function that is growing less rapidly for large error terms than the quadratic function does.

Compare to OLS procedure, robust regression procedures try to devise estimators that are not strongly affected by small deviations from the model assumptions, also it has good efficiency at the assumed model (Huber 1981[2]). Robust regression procedures are also useful when automated regression analysis is required. It will automatically guard against influence of outlying cases in this situation. Numerous robust regression procedures have been developed, such as least median squares, M-estimate, Least trimmed squares. In our study, we have chosen Robust MM-estimate procedure for computing estimates of beta. In this section we descript the robust MM-estimate.

MM-estimate method, introduced by Yohai [3], is a high breakdown and a high efficient estimate method. The general features of robust MM-estimator are:

1.In data-oriented term, robust MM fit is minimally influenced by outliers.

2.In probability-oriented term, the robust fit minimizes the maximum possible bias of the coefficient estimates.

MM-estimator is two-step procedure. First the initial estimate is obtained by S-estimate procedure, and then it is refined by an M-estimate procedure.

1) S-estimate is defined by minimize the dispersion of the residuals. Suppose we have k observations, the initial S-estimate is the value $\hat{\beta}^0$ that:

$$\hat{\beta}^0 = \min_{\beta} s(r_1(\beta), r_2(\beta), \cdots, r_k(\beta)) \quad (4.1)$$

with final scale estimator $\hat{\sigma} = s(r_1(\hat{\beta}^0), r_2(\hat{\beta}^0), \cdots, r_k(\hat{\beta}^0))$ (4.2)

The dispersion is defined as solution of

$$\frac{1}{k}\sum_{i=1}^{k}\rho(\frac{r_i}{s}) = 0.5 \tag{4.3}$$

Where $r_i = \hat{y}_i - y_i \; and \; \hat{y}_i = x_i^T \beta$,

ρ is loss function, it must satisfy the following conditions:

1. ρ is symmetric and continuously differentiable, and ρ(0)=0

2. There exist c >0, such that ρ is strictly increasing on [0,c] and constant on [c,∞)

2) M-estimate, the objective of M-estimate is:

$$\min \sum_{i=1}^{n} \rho(r_i) \tag{4.4}$$

Where r_i is residual, and ρ is symmetric loss function with a unique minimum at zero,

The M-estimate is obtained by solve the equation:

$$\min \sum_{i=1}^{n} \psi(\frac{r_i}{\hat{\sigma}}) x_i = 0 \tag{4.5}$$

Where ψ is derivate of ρ, and $\hat{\sigma}$ is robust scale estimate for residual

The result of robust M-estimate and S-estimate depend on the loss function ρ. In our study we use optimal loss function, introduce in Yohai and Zamar [4]. This is because it has better efficiency and bias control properties. The optional function is:

$$\rho(x) = \begin{cases} 3.25c^2 & if \left|\frac{r}{c}\right| > 3 \\ c^2[1.972 + h_1\left(\frac{x}{c}\right)^2 + h_2\left(\frac{x}{c}\right)^4 + h_3\left(\frac{x}{c}\right)^6 + h_4\left(\frac{x}{c}\right)^8] & if \; 2 < \left|\frac{r}{c}\right| \le 3 \\ \frac{r^2}{2} & if \left|\frac{r}{c}\right| \le 2 \end{cases}$$

Where c is tuning constant and h_1=-0.972, h_2= 0.864, h_3=-0.052, and h_4= 0.002. Figure 2.1 shows the optimal loss function:

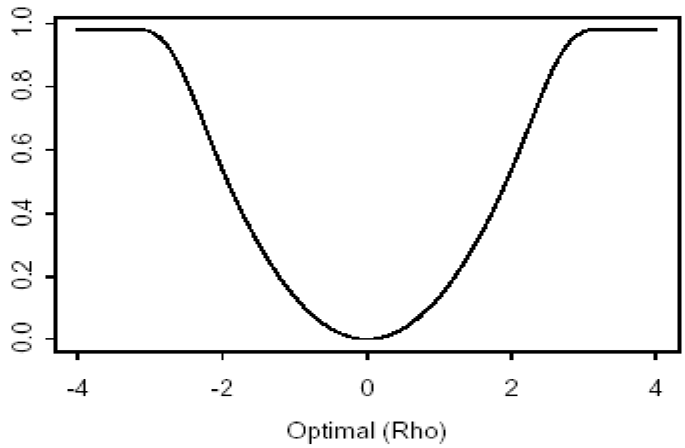

Figure 2.1 Optimal loss function

The MM-estimate method first computes initial S-estimate $\hat{\beta}^0$, then final M-estimate is to find local minimum that is nearest $\hat{\beta}^0$.

For more details on numerical algorithms see Marazzi [4]

2.5 Parameter estimate examples

In these section, we provides two motivating examples of robust estimates, we also compare the result with OLS estimates.

Table 1 displays the result estimations for two companies (LSCC, MEDX) by using Robust MM-estimate method and OLS method. The samples for parameter estimations are quarterly stock data.

Table 1				
Company	LSCC (1999 4th Quarter)		MEDX (1999 2nd Quarter)	
Methods	OLS	Robust MM	OLS	Robust MM
Alpha (intercept)	0.02118898	0.003543713	0.0098	-0.0110
Beta (slop)	-2.180122	2.847663	2.1269	-0.1648
Standard Error	2.4460	0.6931	1.2843	0.8960

Figure 2.2 displays the scatter plots of daily stock returns versus returns for Russell2000 index for the two companies. The solid straight line is the robust fit and dash line is the OLS fit.

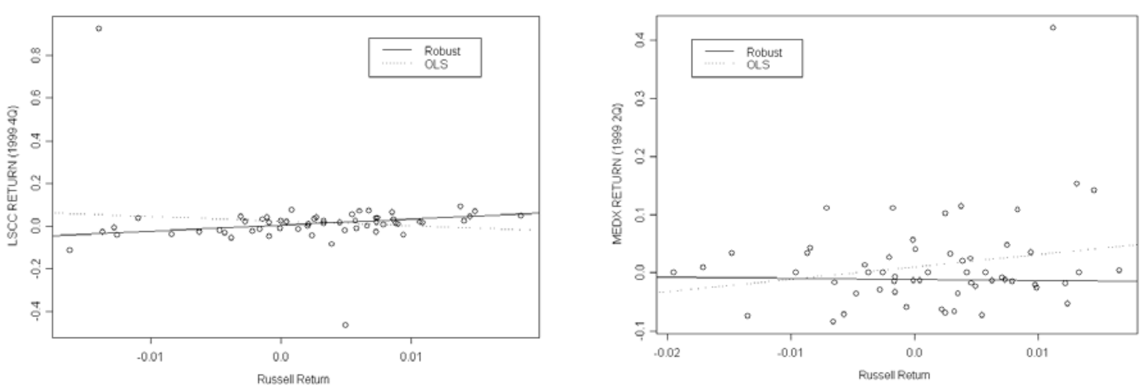

Figure 2.2. OLS and Robust Estimates for Two firms

From Table1, The parameter-based robust regression procedure are significantly different from those based on OLS regression procedure. From Figure 2.2, we can see there is one extreme outlying case in both plots. These extreme outlying cases are leverage points. They involve large residuals and have dramatic effect on the fitted least squares regression function. Since robust MM-estimate procedure dampens the influence of outlying case, as compared to OLS estimation, in an effort to provide a better fit for the majority of cases, the outlying cases had less affect on robust MM-estimator. In LSCC case, the beta estimated by OLS is -2.180122. This indicates the stock return has negative relation with Russell2000 index return. From robust MM-estimate the beta is 2.847663, which implies the stock has about 2.85 times volatile as market and has 2.85 times expected market return. From Figure2.2, It appears that majority data agree with robust estimates. If we remove the outlying case from data, the beta estimated by OLS is 2.838372, which is very close to robust MM-estimate. This example shows that only one outlying case can make significant change in value of OLS estimates and robust MM-estimator provides more desirable estimation. Similar, in MEDX, OLS overestimated the value of alpha and beta due to presence of outliers.

From Table1, our examples also show that outliers can substantially influence, not only the OLS estimates of the betas, but also influence the estimated alphas. Under the single index model, the covariance matrix Σ is computed from betas and required return constraint is base on alphas. Since Σ and alphas are inputs for optimizer, the quality of those estimations will substantially affect the result of optimization procedure, furthermore, it will affect the performance of optimized portfolio. In section 3, we will show how the quality of estimations affects the performance of portfolio.

3. Portfolio optimization:

3.1 Dataset

We download daily history data for all Russell 2000 index components from http://financial.yahoo.com, from 2000 stocks we selected 83 stocks. These stocks satisfy the following requirement: (a) the average trading volume of each stock is above one million shares per trading day. This requirement means that investor will not suffer big bid-ask spreads transaction cost. (b) Each stock has more than a 5-year trading history. This provides us enough history data to compare the performance between different portfolios. Our portfolio includes these 83 stocks.

3.2 Covariance matrix construction and constraints

Under single index model, we can build covariance matrix for the portfolio by using betas, error variances, and market variance. In our study, we use robust-MM regression and OLS regression method to estimate these parameters, then use the results we build the covariance matrix. The covariance matrix and estimated alphas are the inputs for optimization procedure.

To construct the optimized portfolio, the objective function, which has to be minimized, is: $X^T \Sigma X$

In our study, the constraints that must hold for minimizing the objective function are:

1) All money should be fully invested in all stocks

$$\sum_{i=1}^{n} x_i = total\ investment \text{ or } \sum_{i=1}^{n} w_i = 1$$

Where x_i is the amount of money invested in security i, and $w_i = x_i / P$

2) The required daily excess return of portfolio should be not less than 0.1%.

$$\sum_{i=1}^{n} w_i \alpha_i \geq 0.1\%$$

3) No short allowed

$x_i \geq 0 \quad or \quad w_i \geq 0 \quad for\ i = 1...83$

4) The maximum amount of money invested in one stock must less or equal to 5% total portfolio value.

$x_i \leq 5\% \times P \quad or \quad w_i \leq 5\% \quad for\ i = 1...83$

We use these constraints combined with covariance matrix as inputs for portfolio optimization procedure.

3.3 Portfolio performance comparison

We constructed four different portfolios for comparison, they are:

- Portfolio1: Russell 2000 index portfolio
- Portfolio2: Equal weighted portfolio, which invested the equal amount of money into 83 securities.
- Portfolio3: The optimized portfolio based on OLS estimates.
- Portfolio4: The optimized portfolio based on robust estimates.

Among these four portfolios, the portfolio3 and portfolio4 have to be adjusted by quarterly based on quarterly parameter estimations.

To compare the performance among these four portfolios, we divided our comparison to two steps. In the first step, for each stock we estimated the parameters based on the one-quarter data, and then we use optimization procedure to construct the optimized portfolio for this one-quarter data. There is no prediction procedure involved in this step. The portfolio3 and portfolio4 are constructed based on the realized OLS estimates and robust estimates respectively. The reason behind this is that many available prediction methods can be chosen, and such prediction procedures might have unexpected effects on the results. In second step, we predict the parameters based on history data, and then use them as optimizer inputs to construct the optimized portfolios. We describe the procedure and result for each step in following section.

In the first step, we selected all stock daily return data from 1/2/2000 to 12/31/2003. The data has been divided by quarterly, totally 16 quarters for each stock. For each quarter, we estimate alphas, betas, and error variances for all stocks by using Robust MM-estimate procedure and OLS procedure. Also the quarterly variance of Russell2000 index has been computed. We compute the covariance matrixes based on these result. Since there are 83 stocks in portfolio, the covariance matrix is 83×83 matrix for each quarter. Now we have all inputs for portfolio optimization procedure. By using portfolio optimization procedure, the Portfolio3, and Portfolio4 are constructed. The optimized portfolio is only used for current quarter. Based on the output, the portfolio performance and risk will be easily computed. We repeat same procedure for all 16 quarters. The initial investment for next quarter is base on the last day portfolio value of current quarter. Since we need adjust portfolio quarterly, the transaction costs, such as brokerage fee, bid-ask spreads, are considered in our study. We assume the transaction costs will be 1% of the total amount of money that involves in adjusting portfolio. This cost will be abstracted from portfolio at beginning of each quarter. The Figure 3.1 displays the portfolio value during four-year period.

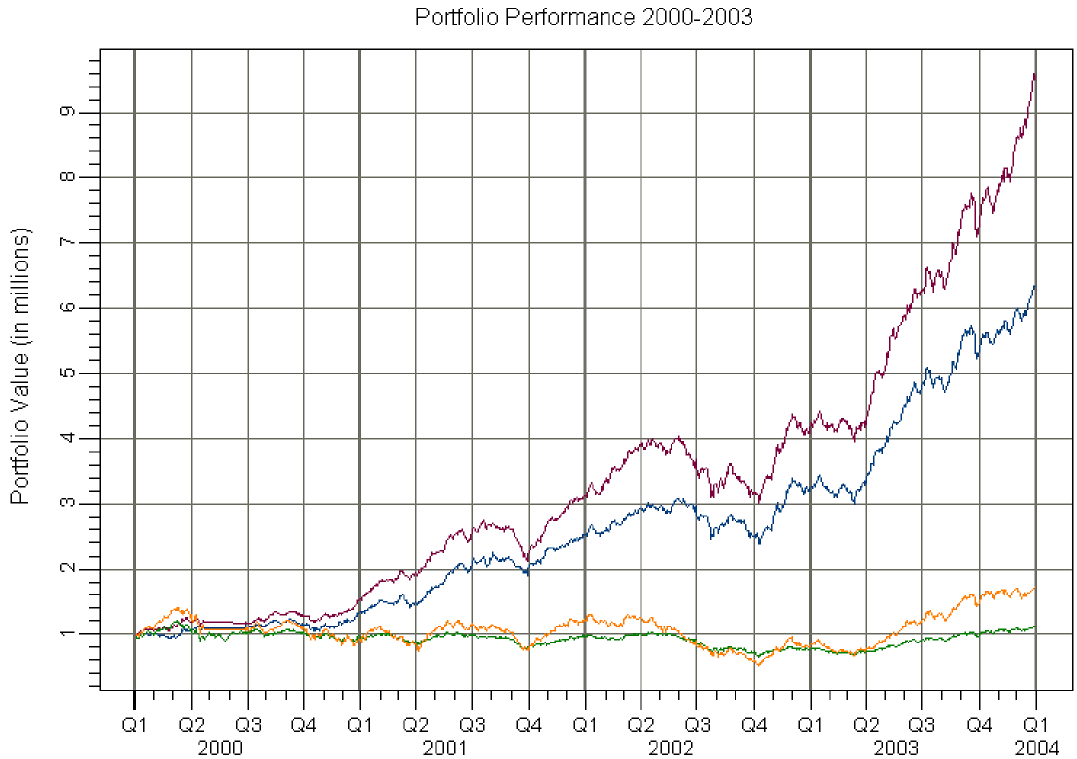

Figure 3.1

Green Line: Portfolio1. Orange Line: Portfolio2. Blue Line: Portfolio3. Purple Line: Portfolio4

The Figure3.2 displays quarterly risk for four portfolios during the 4-year period

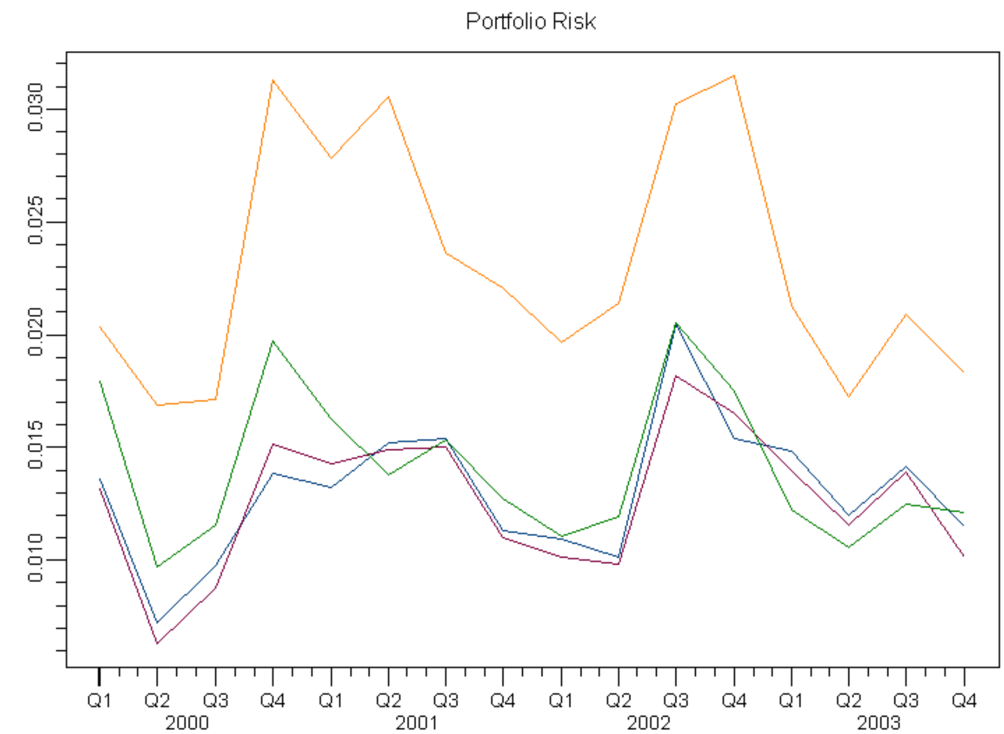

Figure 3.2 Portfolio Risk

Green Line: Portfolio1. Orange Line: Portfolio2. Blue Line: Portfolio3. Purple Line: Portfolio4

Figure3.2 shows that equal weighted portfolio has highest risk over four-year time period. The risks for Portfolio1, Portfolio3, and Portfolio4, are very close. This implies we didn't suffer higher risks in order to get higher returns. From Figure3.1 and Figure 3.2, during 4-year period, the Portfolio4 has the best performance. At end of 2003, the total portfolio value of Portfolio1, Portfolio2, Portfolio3, and Portfolio4 were $1103338.3, $1693959.2 $ 6306223.9, and $9487728 respectively.

At this point, we see that the robust MM-estimate provides more reliable estimates than OLS for history data. However, in the real world, there are more people interested in estimator prediction. Next step of our study is to predict betas, alphas, and error variances base on history estimates.

To construct a predicted covariance matrix, we need enough history betas, and error variances. For each stock we chose 32-week history data to predict the beta, alpha, error variance for next quarter. For example, if we want forecast beta for 1st quarter of 2000, we choose daily return data from 5/21/1999 to 12/31/1999 (32weeks) as history data. We use moving window method to get sequence parameter estimations. Our window size is12 weeks long, each time robust MM-estimate and OLS procedure have been used to estimate betas and error variances from daily return data that inside the window, after that, we move window forward by one week, and estimate betas and error variances. We keep moving window until we reach the end of history data. Since each time only small portion of data has been changed, this then will help us to capture the trend of estimated parameters. By using moving window method on 32 weeks data, we will get 20 group estimated parameters.

For each stock we predict 16 quarterly betas, alphas, and error variances by using history data; also, we use same method to predict the market variance. Based on all these predictions, the covariance matrix is constructed. We use portfolio optimization procedure to construct optimized portfolio. As we did in the first step, we compute portfolio values, portfolio risks for each portfolio during 4-year period. Figure 3.3 displays the portfolio values for four portfolios.

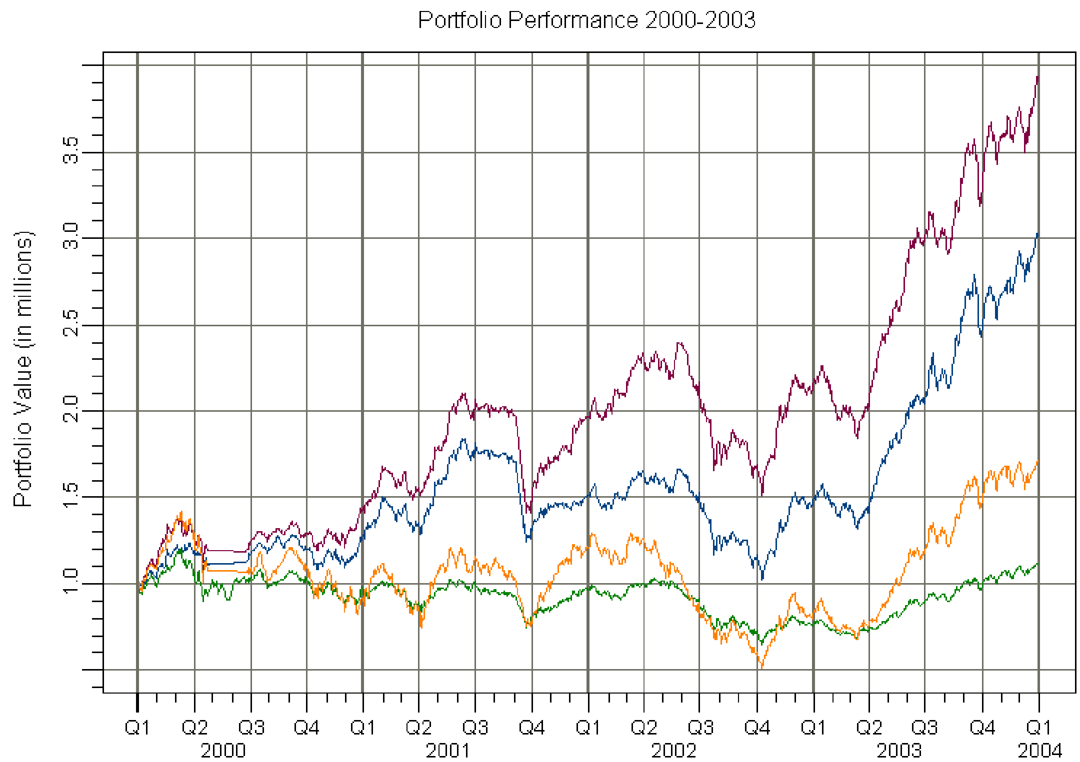

Figure 3.3

Green Line: Portfolio1. Orange Line: Portfolio2. Blue Line: Portfolio3. Purple Line: Portfolio4

From Figure 3.3, we can see the result is consistent with result in Figure 3.1. At end of a 4-year period, the total portfolio value of Portfolio4, Portfolio3, equal weighted portfolio and Russell2000 portfolio are $ 3897492.5, $3000923.5, 1693959.2 and 1103338.3 respectively. The transaction cost also been considered in portfolio calculation. Figure 3.4 displays the risks associated with each portfolio.

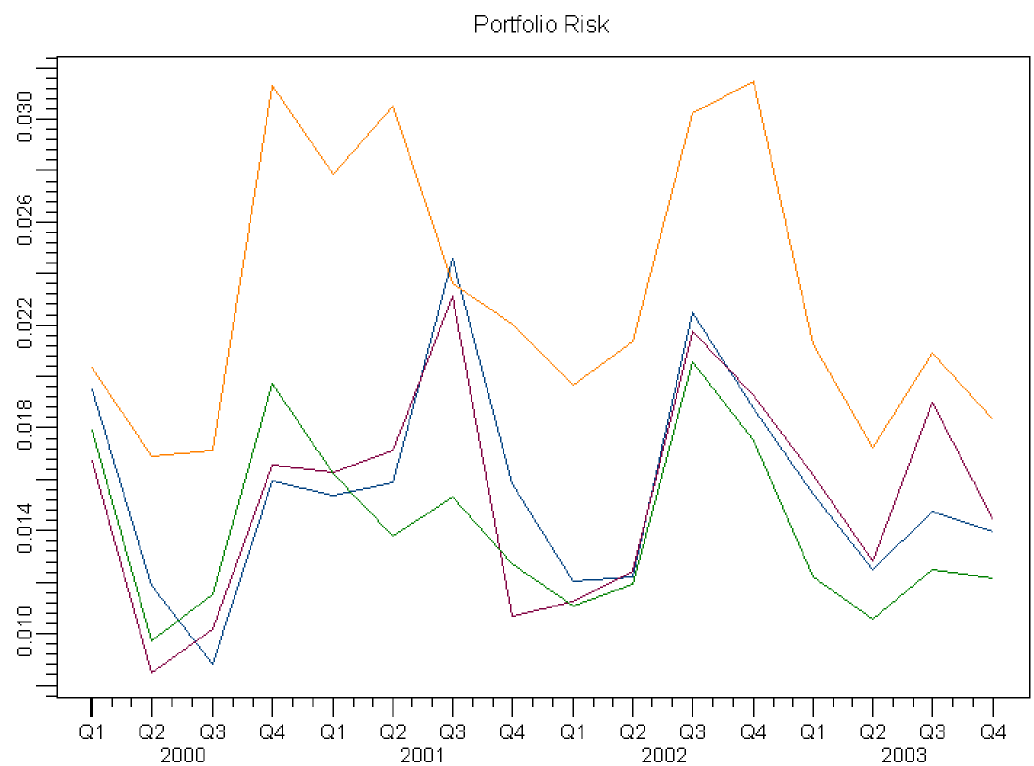

Figure 3.4 Portfolio Risk

Green Line: Portfolio1. Orange Line: Portfolio2. Blue Line: Portfolio3. Purple Line: Portfolio4

From Figure 3.4, the risk for Russell2000 and equal weighted portfolio are the same as they are in Figure3.2. The portfolio risk for the robust portfolio and OLS portfolio are higher than they are in Figure3.2, but they are still close to each other and close to market risk level. Also, The biggest difference between Figure3.2 and Figure3.4 is in 3rd quarter of 2001. This is because September 11th tragedy happened in that Quarter. In Figure 3.2, the optimized portfolios were based on realized parameters, so the optimized portfolios only included the stocks that are not sensitive to September 11th event. In Figure 3.4, the optimized portfolios were based on predicted parameters. Since it is impossible to predict

September 11^{th} event, so the optimized portfolio included some stock that are sensitive to the tragedy. This is the reason why in that quarter the risks of two optimized portfolio were significantly increased.

Based on portfolio performance analysis, the optimized portfolio based on robust regression estimates is the most efficient portfolio among four portfolios. Since the performance of optimized portfolios directly reflect the quality of the estimated parameters, which are the inputs of optimization program, so our study conclude that for small capitalization firms, the robust regression procedure is the preferred procedure for alpha, beta, error variance estimations.

4. Summary

The robust regression technology is relatively new statistical technology. The research and implementation of the robust regression in financial industry also are very new topics. The goal of our study is to investigate the use of robust estimation procedures in conjunction with portfolio optimization. The efficient optimization algorithm exists and has been implemented in different software packages. The quality of the results depends on the quality of inputs of the optimization program. To provide reliable parameter estimations is the extremely important task for portfolio manager. Based on the consistent results from our portfolio performance analysis, we suggest that for small cap companies, the robust regression procedure provides more desirable parameter estimations than ordinary least squares regression procedure does. Our study also suggests that the stock selection process can be designed in a systematic fashion.

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Appendix.

The stock tickers for 83 stocks:

AAI	AAII	ABGX	ACAI	ADIC	ADPT	AEOS	AKS	AMR	ANDW	ARRS	ASYT
ATVI	AYE	BEV	BJ	BORL	BRKS	CAL	CBB	CCK	CCUR	CDE	CMGI
CMOS	CMS	CNET	CRAY	CRUS	CVTX	CYMI	CYTC	ELNK	ERES	GNSS	GNTA
GT	GTI	GTW	GW	HL	HLIT	HLYW	IGL	KLIC	LGND	LPX	LSCC
MANU	MEDX	MENT	MESA	MGAM	MTON	MWY	NITE	NKTR	NWAC	OSIP	PMTC
PRTL	PSUN	PWAV	RCNC	REMC	RFMD	RGEN	SAPE	SCHN	SCON	SFE	SGI
SMTC	SRP	SSTI	SWKS	TQNT	TTWO	VRSO	VTSS	WWCA	YELL	ZRAN	